SQIsign

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https://sqisign.org/

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Short

Public Key	Signature	Security
66	148	NIST-1
98	222	NIST-3
130	294	NIST-5

Quaternion and **I**sogeny

Only candidate based on isogenies of supersingular elliptic curves

signature

Fiat-Shamir paradigm

Weren't isogenies broken?

Yes, SIKE was broken:

- Security based on isogeny problem with torsion point information;
- Very efficient classical algorithm to solve the SIKE problem found in 2022 (Castryck–Decru, Maino–Martindale, Robert);
- SQIsign v2 incorporates techniques developed in these attacks.

But isogenies live:

- The supersingular isogeny problem still is exponentially difficult for quantum computers;
- SQIsign's EUF-CMA reduces almost exactly to the supersingular isogeny problem;
- Breaks are part of life in cryptography.

Isogenies = algebraic group morphisms of elliptic curves

$$y^2 = x^3 + x \longrightarrow y^2 = x^3 + 1$$

Isogenies = algebraic group morphisms of elliptic curves

$$y^2 = x^3 + x \xrightarrow{\quad ?? \quad} y^2 = x^3 + 1$$

The isogeny problem

Given two isogenous curves, compute an isogeny between them.

For random supersingular curves defined over a finite field $\mathbb{F}_{p^2}$:

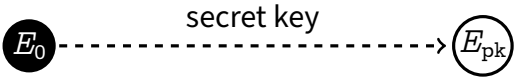
- Best classical attack in $p^{1/2+\epsilon}$ — Delfs–Galbraith, DCC 2016
- Best quantum attack in $p^{1/4+\epsilon}$ — Biasse–Jao–Sankar, Indocrypt 2014

Endomorphisms = Isogenies from a curve to itself

The supersingular endomorphism ring problem

Given a supersingular curve, compute a basis for its endomorphism ring

- Equivalent to the supersingular isogeny problem under (mostly tight) polynomial-time reductions
- Best algorithms = same as for the isogeny problem

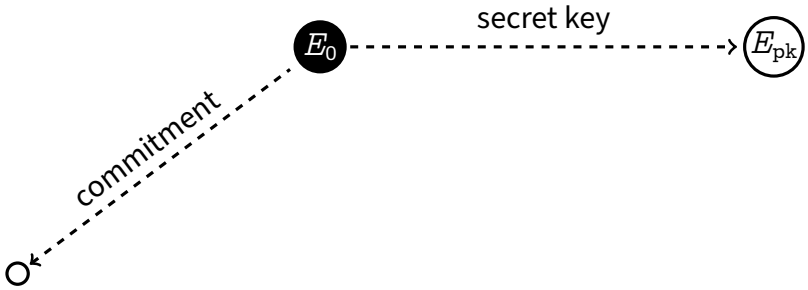


● Fixed curve: $y^2 = x^3 + x$

Legend:

⟶ public isogeny ⇢ secret isogeny

SQIsign's sigma-protocol



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Legend:

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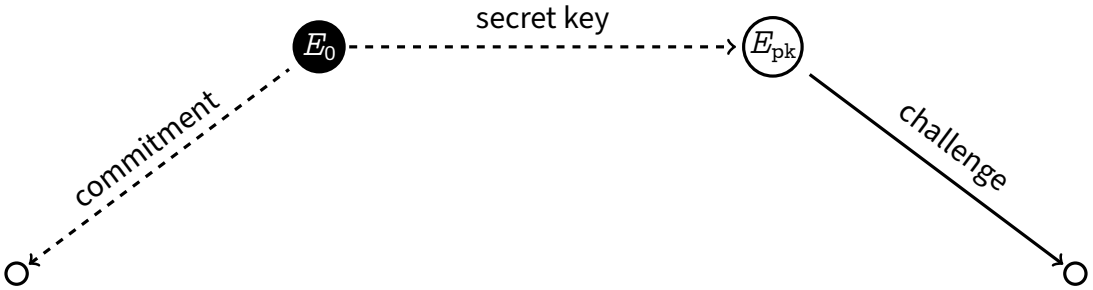
SQIsign's sigma-protocol



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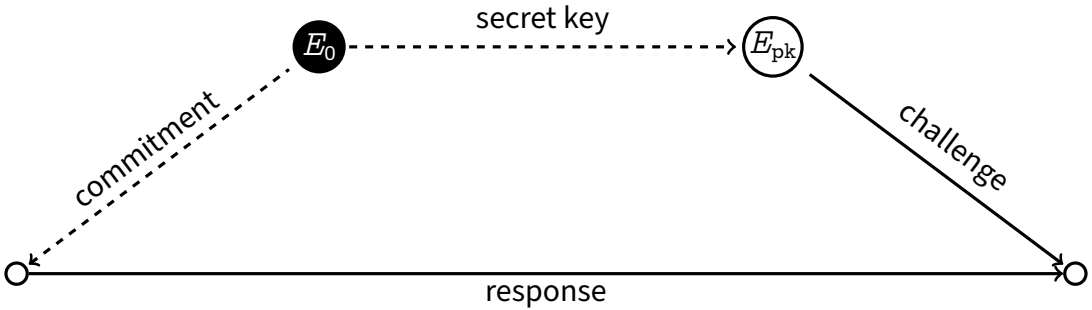
SQIsign's sigma-protocol



● Fixed curve: $y^2 = x^3 + x$

Legend:

⟶ public isogeny ⇢ secret isogeny

Single choice: characteristic of the finite field

Constraint: $p + 1$ divisible by a large power of 2, for performance reasons

p	Security
$5 \cdot 2^{248} - 1$	NIST-1
$65 \cdot 2^{376} - 1$	NIST-3
$27 \cdot 2^{500} - 1$	NIST-5

All other pre-computed constants are easily explainable and provably do not affect security

Parameter set	KeyGen	Sign	Verify
Reference implementation (with default GMP installation)			
NIST-I	71.8	163.1	11.3
NIST-III	188.2	427.0	30.4
NIST-V	325.4	751.8	61.9
Reference implementation (with GMP `--disable-assembly`)			
NIST-I	84.4	203.1	11.3
NIST-III	227.9	548.9	30.5
NIST-V	402.6	1021.0	62.2
Assembly-optimized implementation for Intel Broadwell or later			
NIST-I	43.3	101.6	5.1
NIST-III	134.0	309.2	18.6
NIST-V	212.0	507.5	35.7

Performance in 10^6 CPU cycles on an Intel Core i7-13700K CPU.

Changes from Round 1

Before

“1D” paradigm
(De Feo–Kohel–Leroux–Petit–Wesolowski 2020)

Slow keygen and signing

Reduction to non-standard problem

More complex algorithms

After

“HD” paradigm
(Dartois–Leroux–Robert–Wesolowski 2023)

Fast

Reduction to “well-studied” problem

More complex mathematics

Endomorphism ring with hints (Aardal–Basso–De Feo–Patranabis–Wesolowski 2025)

Given:

- a random supersingular curve E,
- a polynomially-sized list of random isogenies of E of degree $< \sqrt{p}$,

compute the endomorphism ring of E.

Endomorphism ring problem

Given a random supersingular curve E, compute its endomorphism ring.

Endomorphism ring with hints (Aardal–Basso–De Feo–Patranabis–Wesolowski 2025)

Given:

- a random supersingular curve E,
- a polynomially-sized list of random isogenies of E of smooth degree $< \sqrt{p}$,

compute the endomorphism ring of E.

Endomorphism ring problem

Given a random supersingular curve E, compute its endomorphism ring.

“Qlapoti: Simple and Efficient Translation of Quaternion Ideals to Isogenies”
Borin–Corte-Real Santos–Komada Eriksen–Invernizzi–Mula–Schaeffler–Vercauteren

- Simplifies key subroutine of SQIsign;
- Total speed-up between 10% and 80%;
- Reduced memory footprint;
- Removes some heuristics from security proof.

Summary

SQIsign advantages:

- Very small public keys and signatures (< RSA)
- Good performance
- Conservative assumption
- Still some potential for speed-ups
- Adds diversity to pq-signature portfolio

SQIsign disadvantages:

- 1-2 orders of magnitude slower than lattices ($\approx$ SPHINCS+)
- New assumption
- Constant-time implementation still a research topic