Isogeny-based Crypto
Coming of Age

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When does one become an adult?

A brief history of ECC

1976 Diffie–Hellman key exchange

1977 RSA

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1985 Miller and Koblitz introduce elliptic curves to cryptography

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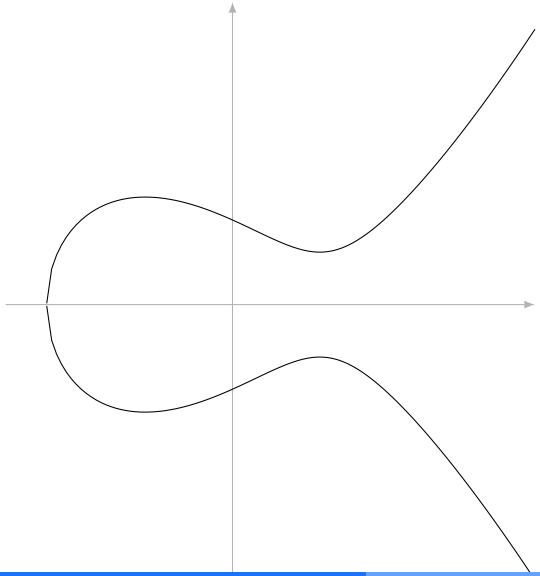
2017 NIST calls for post-quantum algorithms

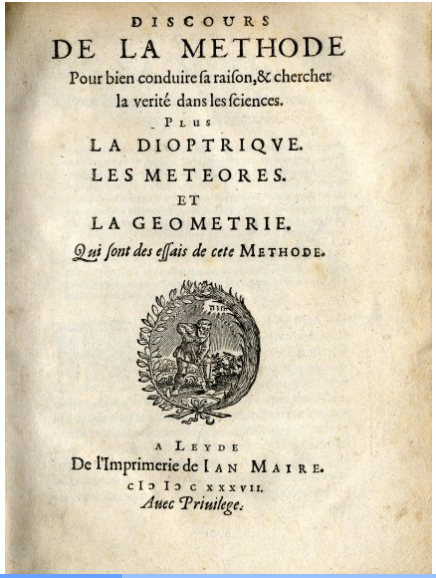
A brief history of ECC

1976 Diffie–Hellman key exchange
1977 RSA
1985 Miller and Koblitz introduce elliptic curves to cryptography
1994 Shor's quantum algorithm
1997 Couveignes' isogeny-based key exchange
1999 NIST standardizes its first elliptic curves
2017 NIST calls for post-quantum algorithms

High school isogenies

Geometry

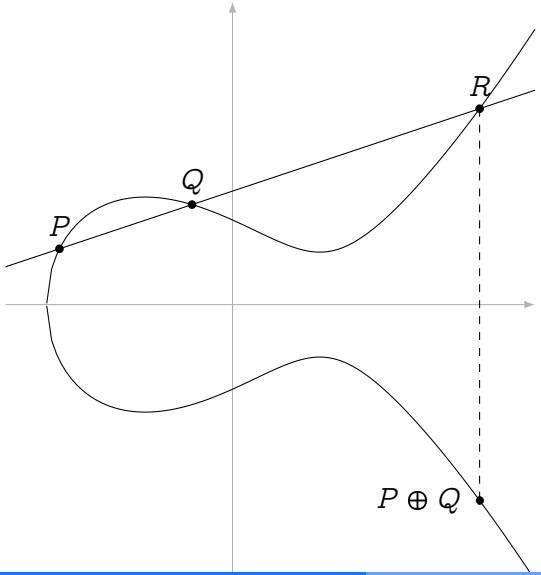




DISCOURS
DE LA METHODE
Pour bien conduire sa raison, & chercher
la verité dans les sciences.
Plus
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui sont des essais de cette METHODE.

A LEYDE
De l'Imprimerie de IAN MAIRE.
clɔ Iɔ c xxxvii.
Avec Privilege.

Groups


R
Q
P
P ⊕ Q


REPUBLIQUE FRANÇAISE
Evariste Galois
1811-1832
postes 1984
2,10+0,40
révolutionnaire et géomètre


NORGE
NIELS HENRIK ABEL
5 50
1802-1829

Isogenies

Algebraic morphisms: $\left(\dfrac{5x^2 + 5x + 4}{x + 1}, \quad y \dfrac{2x^2 + 4x - 4}{x^2 + 2x + 1} \right)$

Group morphisms: $\varphi(P + Q) = \varphi(P) + \varphi(Q)$

Isogenies

Algebraic morphisms: $\left(\dfrac{5x^2+5x+4}{x+1}, \quad y\dfrac{2x^2+4x-4}{x^2+2x+1}\right)$

→ Interpolation

Group morphisms: $\varphi(P+Q) = \varphi(P) + \varphi(Q)$

→ Groups **of** morphisms

Degree

$$\deg(\varphi) \in \mathbb{N}^+$$

- A measure of “size”;

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- Multiplicative;

$$\deg(\psi \circ \varphi) = \deg(\psi)\deg(\varphi)$$

Degree

$$\deg(\varphi) \in \mathbb{N}^+$$

- A measure of “size”;
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$$\deg(\psi \circ \varphi) = \deg(\psi)\deg(\varphi)$$

- A measure of “complexity”:

  only isogenies of small degree fit into a computer!

The isogeny problem*

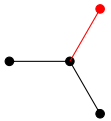
*Decisional problem **is easy**.

The isogeny problem*

*Decisional problem **is easy**.

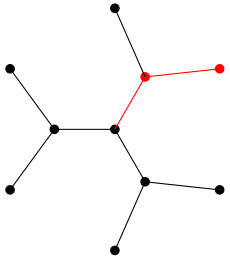
Isogeny graphs

degree 2



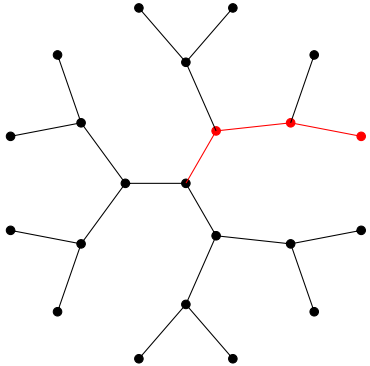
Isogeny graphs

degree 4



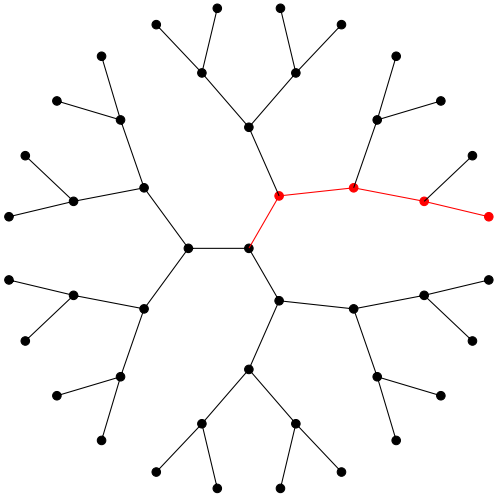
Isogeny graphs

degree 8



Isogeny graphs

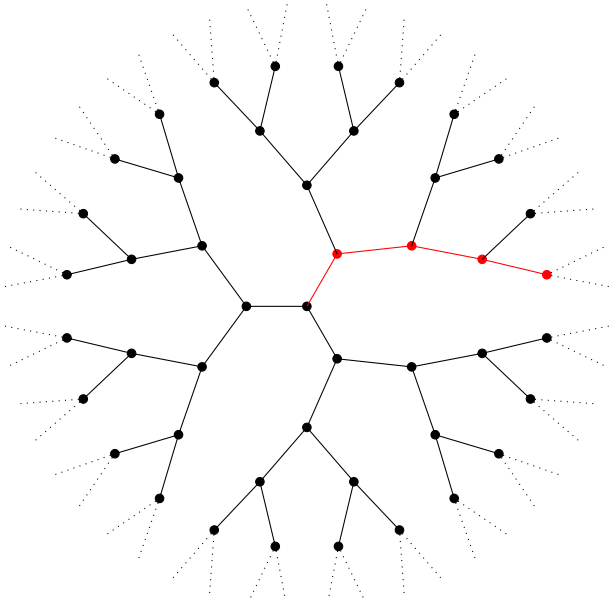
degree 16



Isogeny graphs

degree 2^x

$\rightarrow$ Isogeny **path** problem

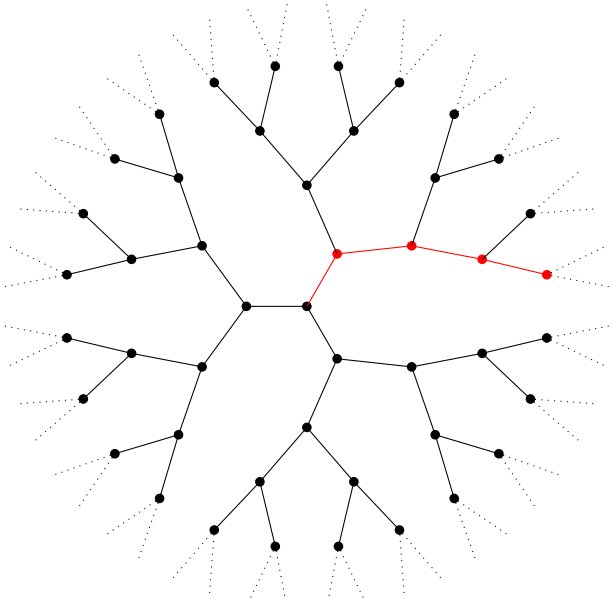


Isogeny graphs

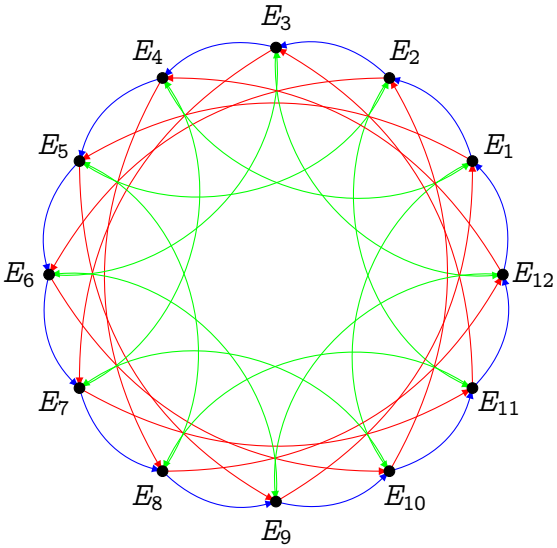
degree 2^x

$\rightarrow$ Isogeny **path** problem

$\rightarrow$ Hash functions
(Charles, Goren, Lauter '09)

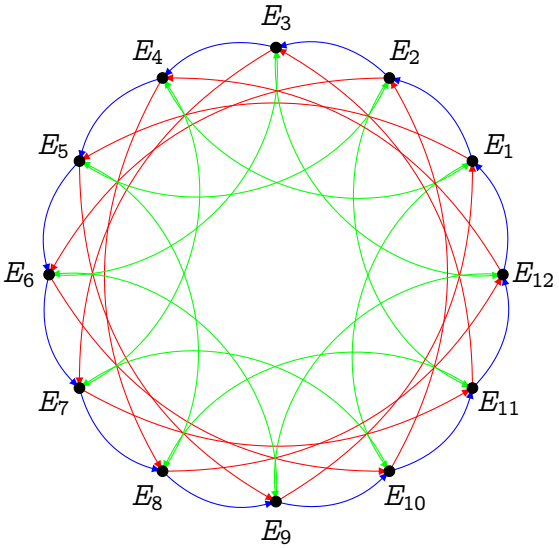


Couveignes ’97-’06 / Rostovtsev–Stolbunov ’06 / CSIDH ’19



Action $E_2 = E_1^{\bullet}$
$E_5 = E_1^{\bullet}$
$E_{10} = E_1^{\bullet}$

Couveignes ’97-’06 / Rostovtsev–Stolbunov ’06 / CSIDH ’19



Group $((E^{\bullet})^{\bullet})^{\bullet} = E^{(\bullet\bullet\bullet)} = E_1^{\bullet}$

Action $E_2 = E_1^{\bullet}$
$E_5 = E_1^{\bullet}$
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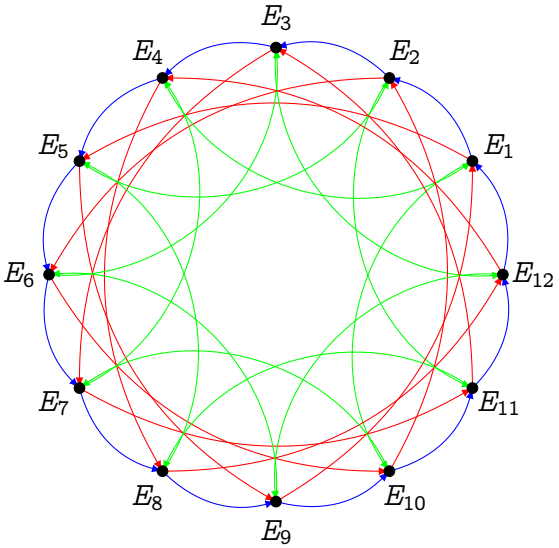
Couveignes ’97-’06 / Rostovtsev–Stolbunov ’06 / CSIDH ’19



Commutative $E^{(\bullet\bullet)} = E^{(\bullet\bullet)}$

Group $((E^{\bullet})^{\bullet})^{\bullet} = E^{(\bullet\bullet\bullet)} = E_1^{\bullet}$

Action $E_2 = E_1^{\bullet}$
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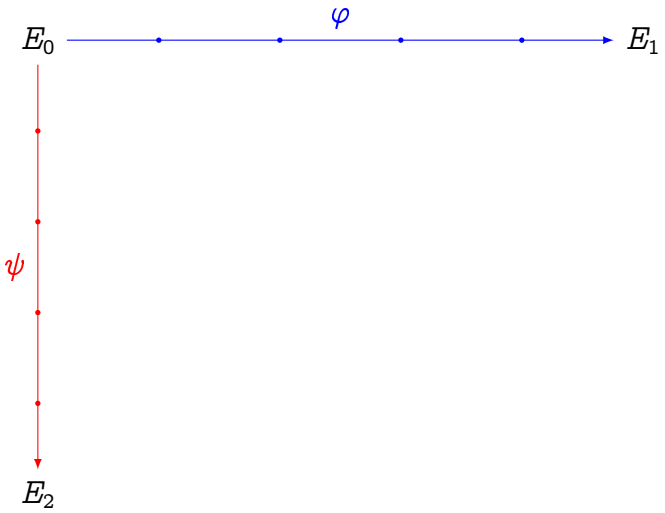
SIDH ’11 / SIKE ’17



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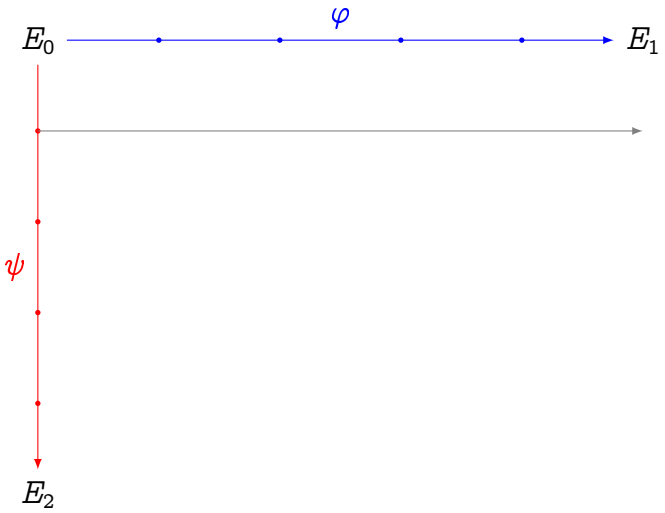

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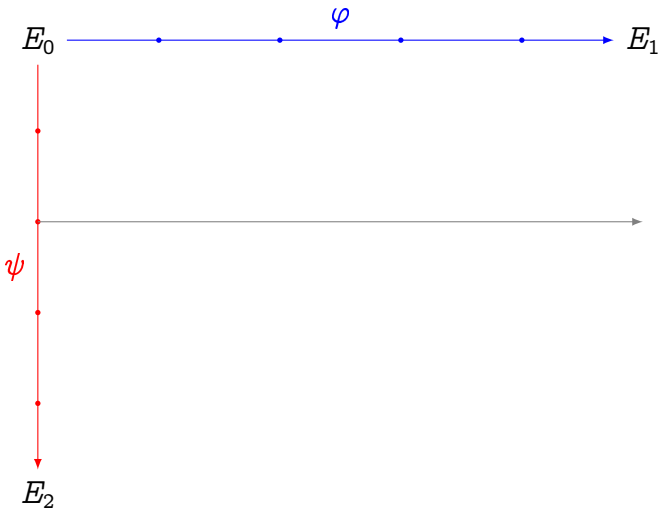

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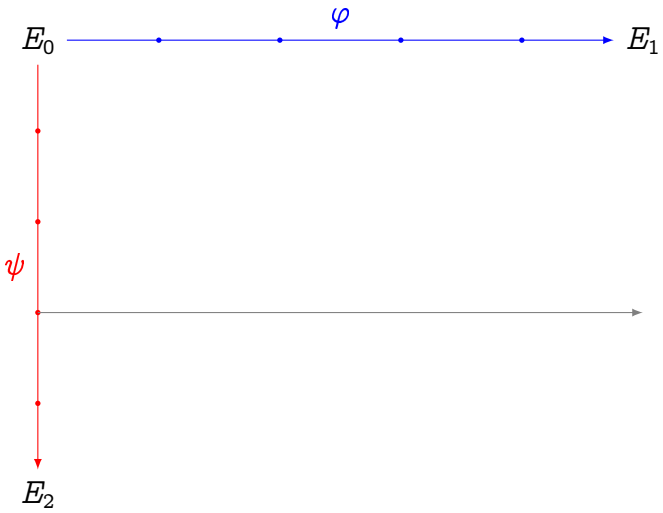

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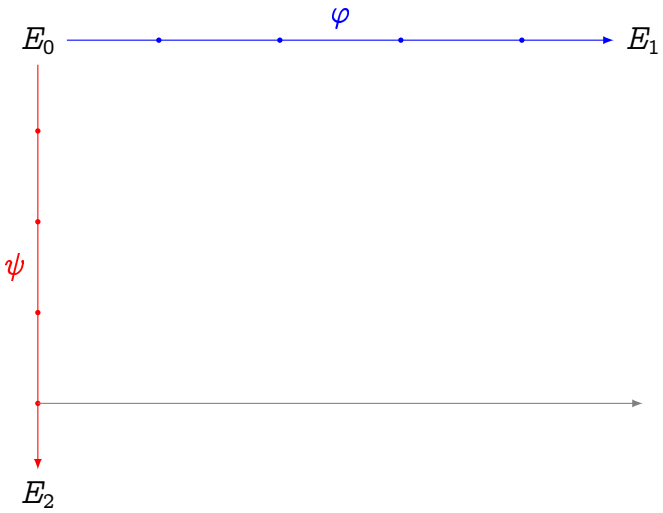

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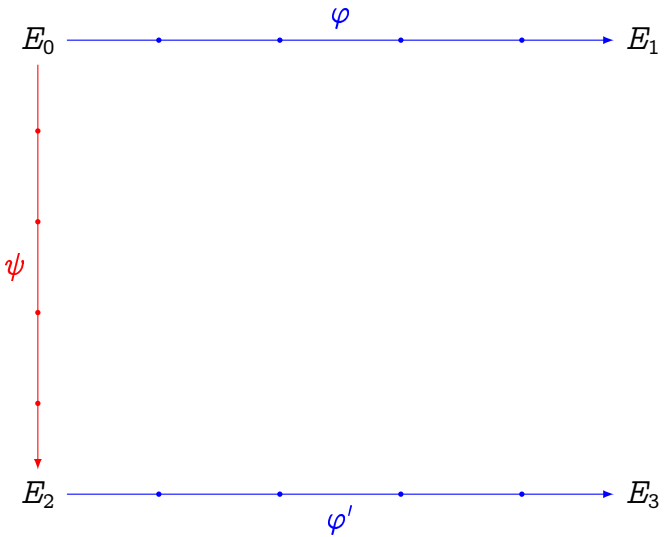

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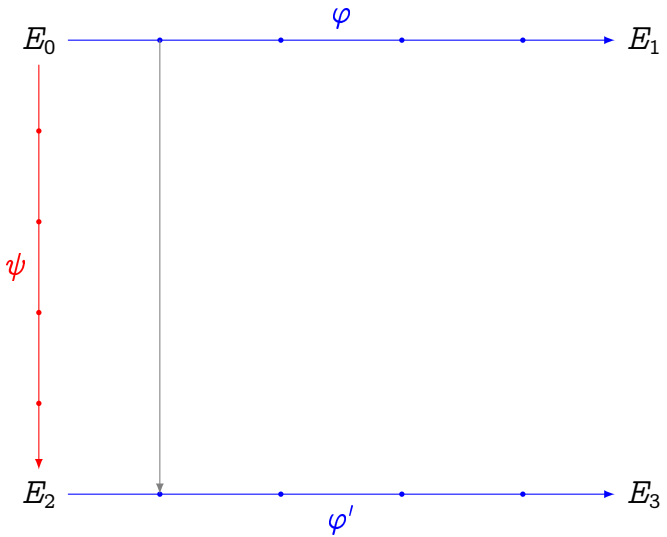

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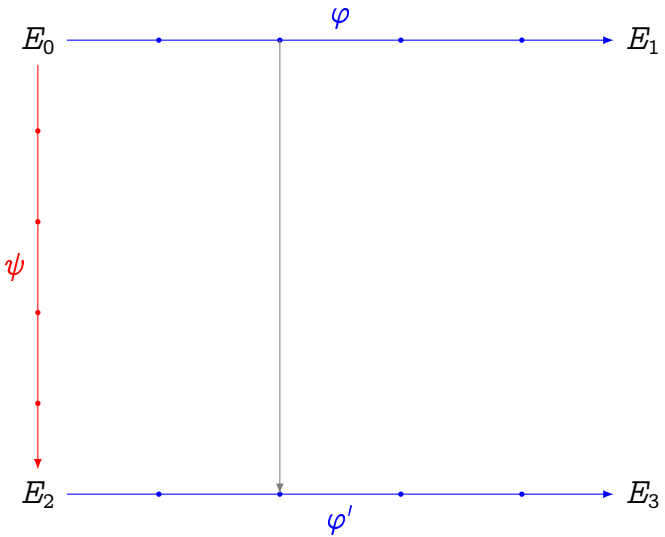

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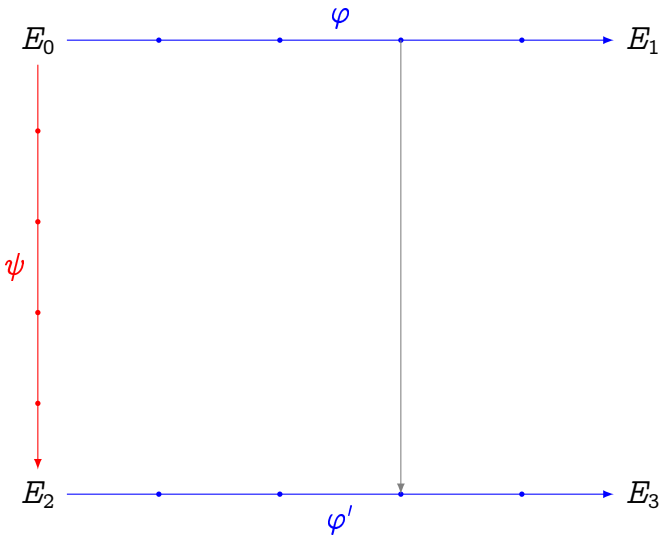

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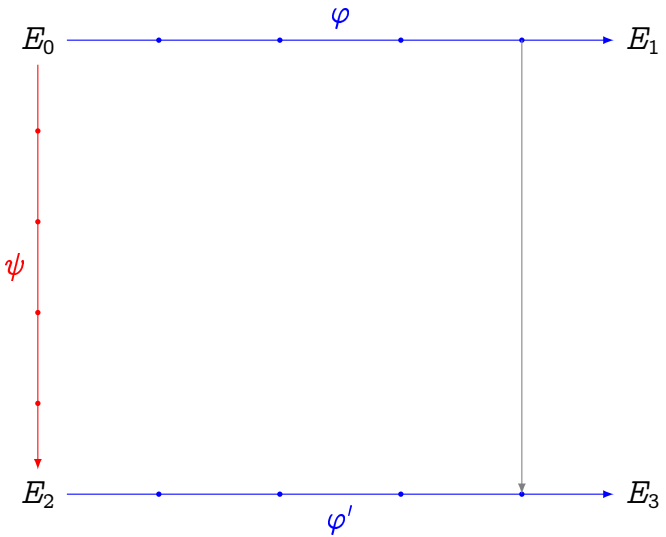

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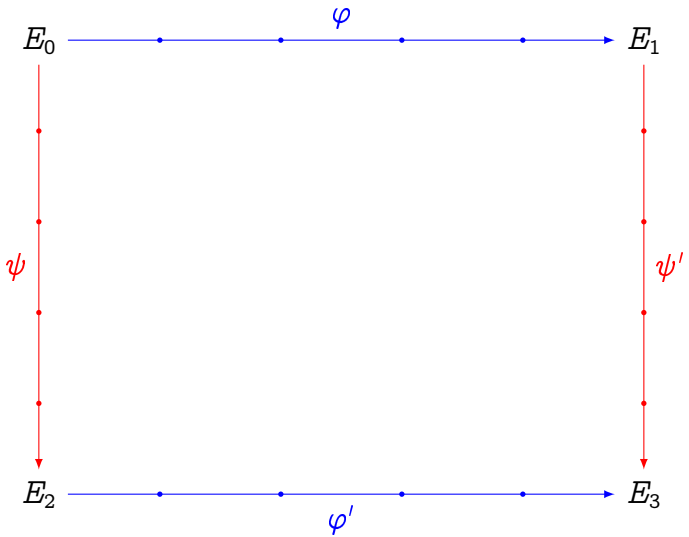
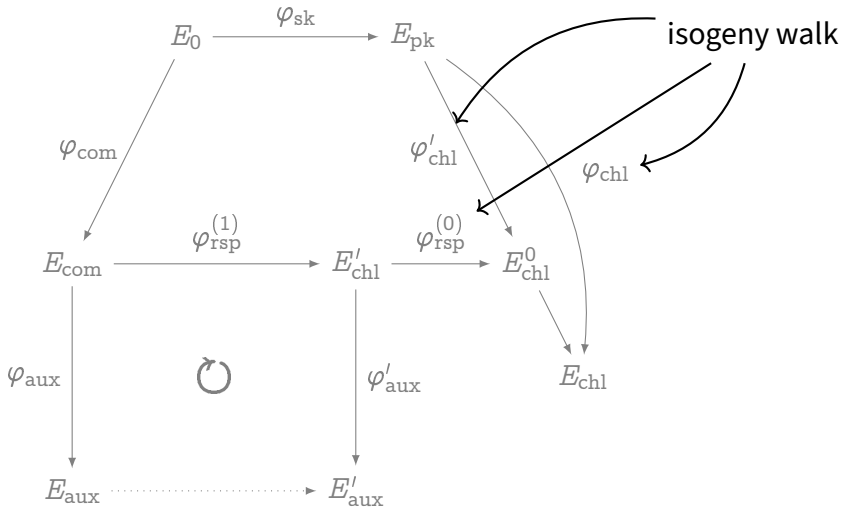


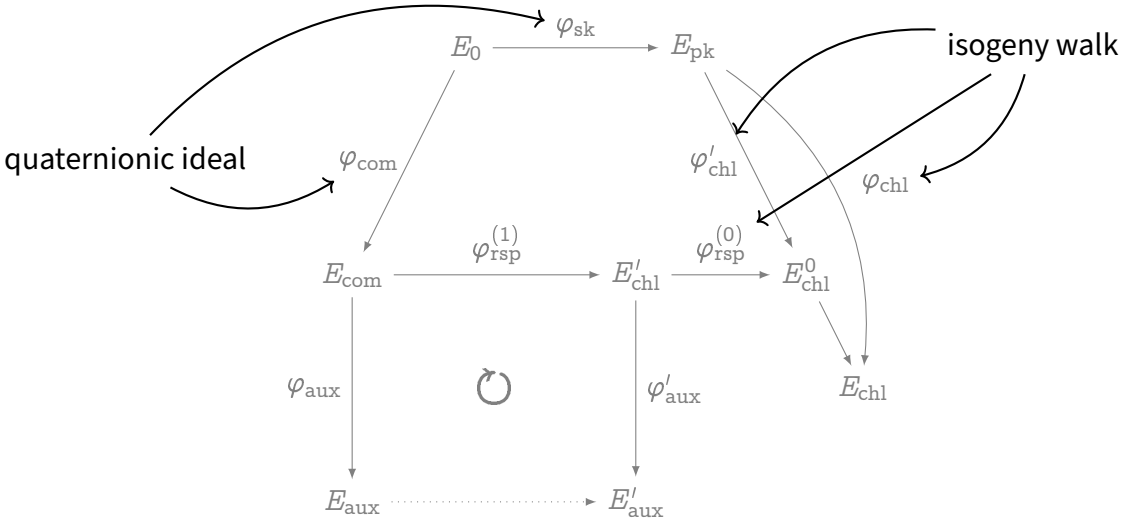

AND THEN THEY SAID...
OUR KEY EXCHANGE IS QUANTUM-SAFE!

SQisign '20



[1] From *"SQIsign2D–West: The Fast, the Small, and the Safer"*, Asiacrypt 2024.

SQisign '20



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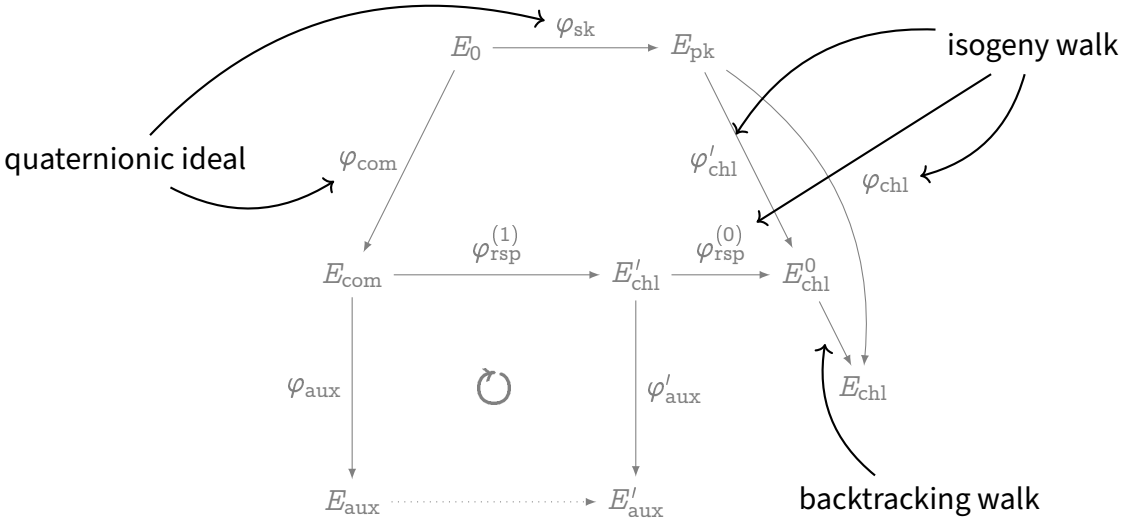

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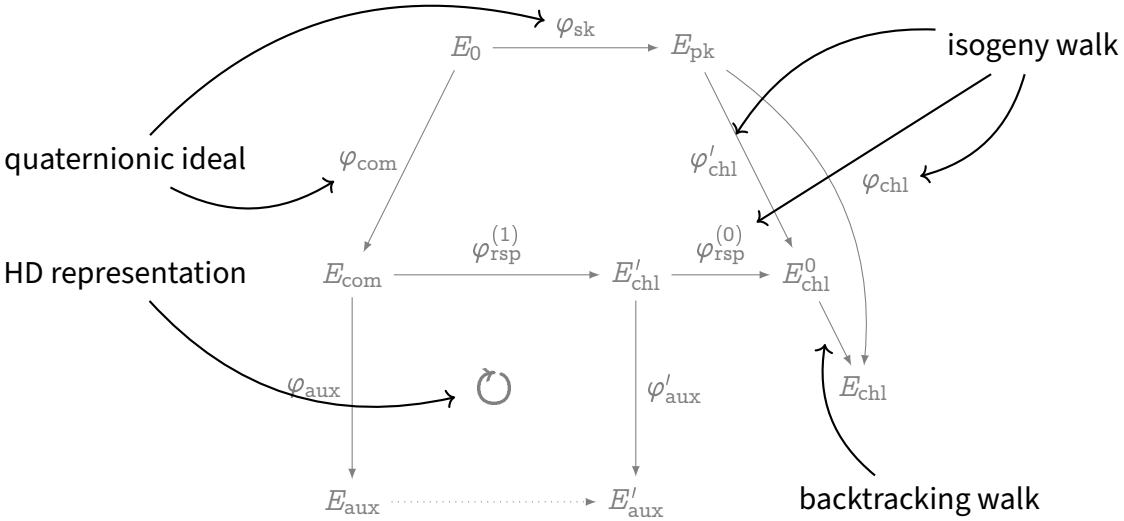
SQisign ’20



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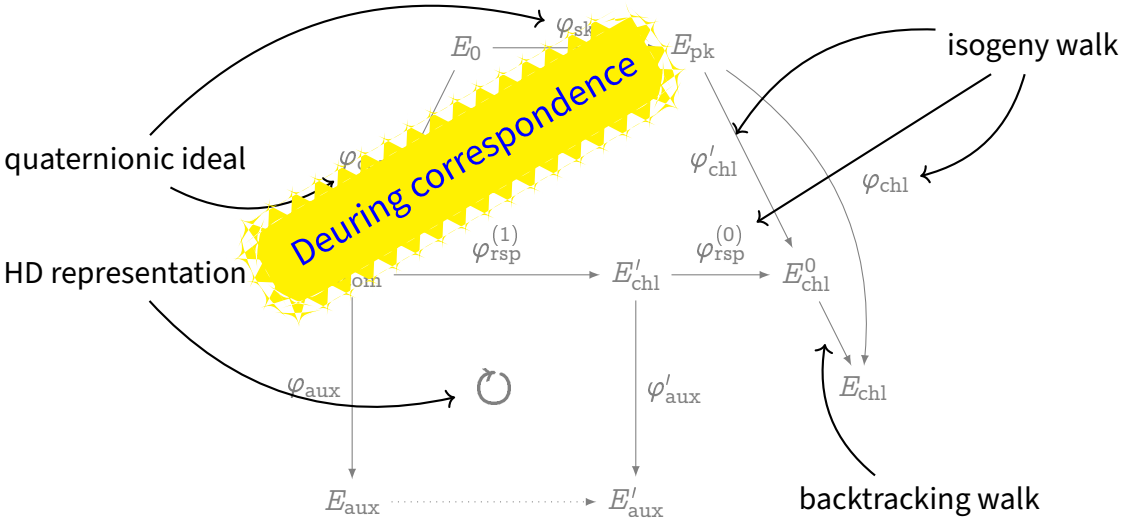

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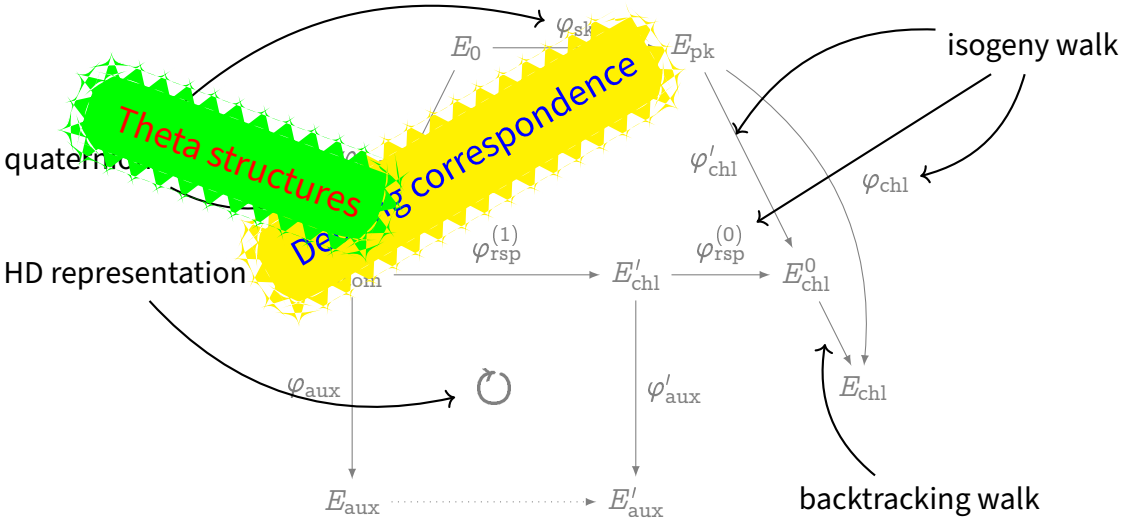

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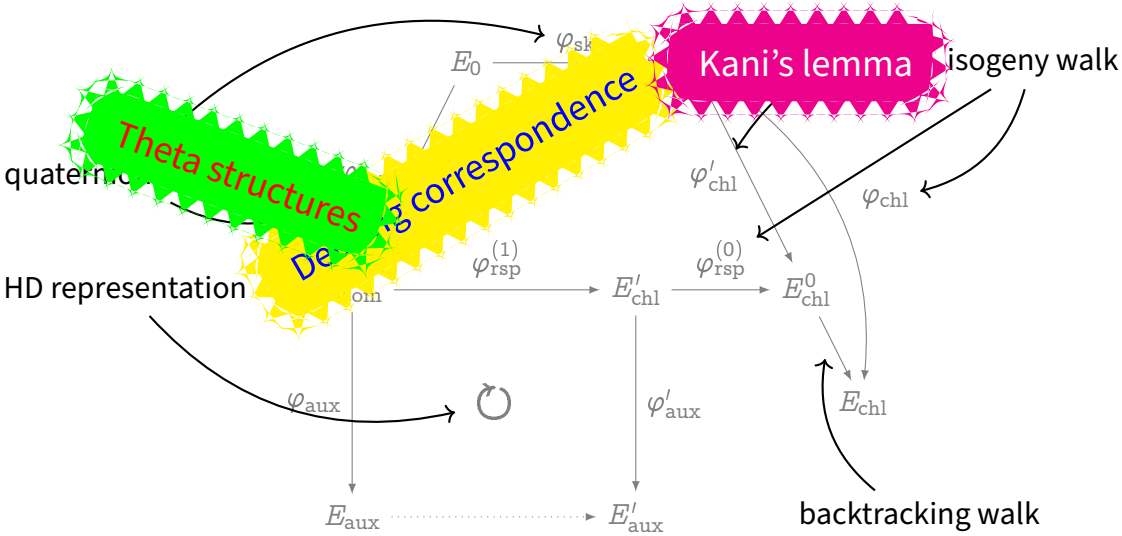

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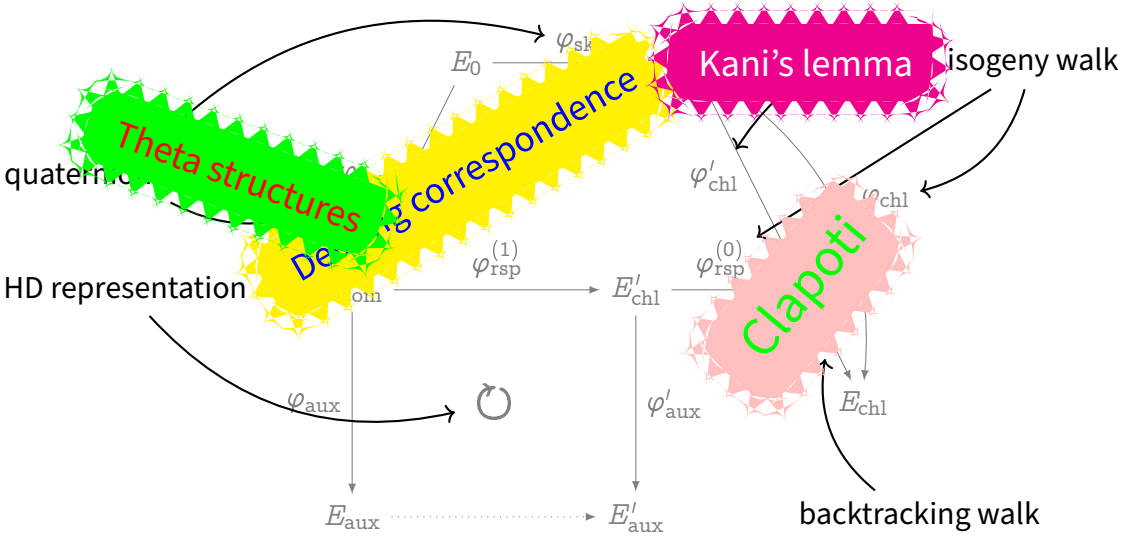

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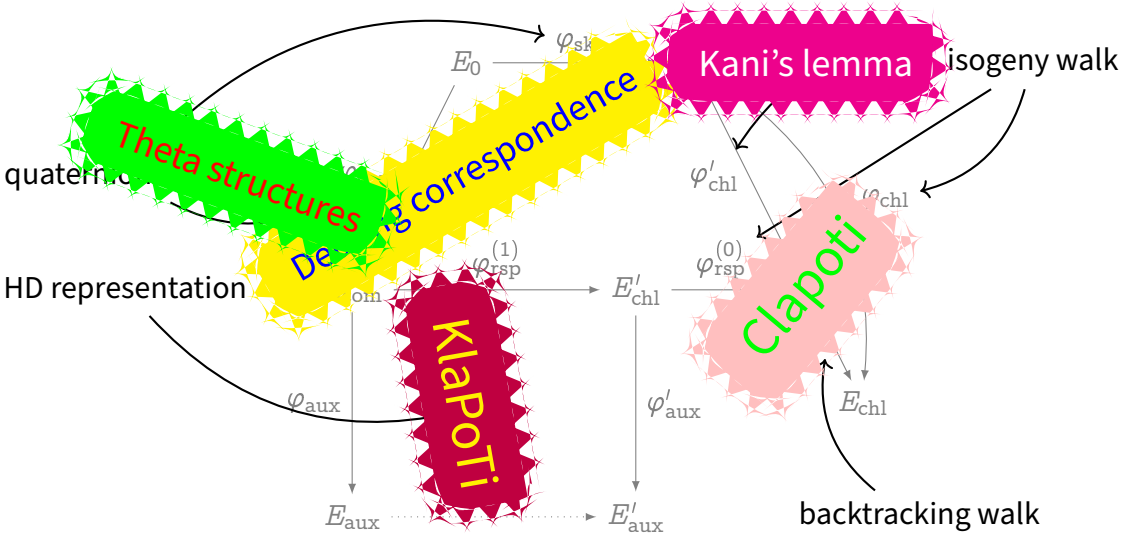

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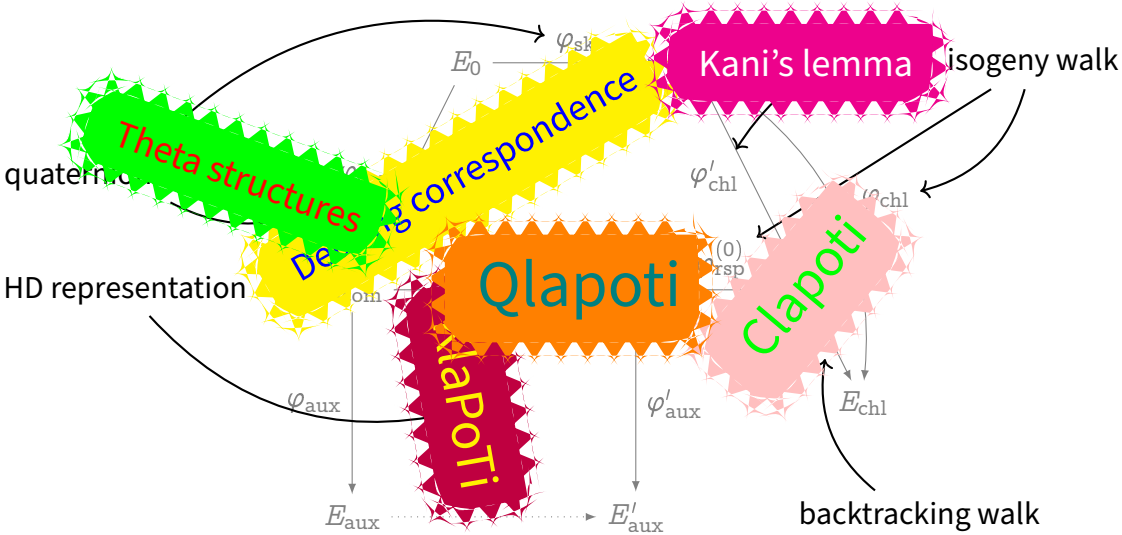

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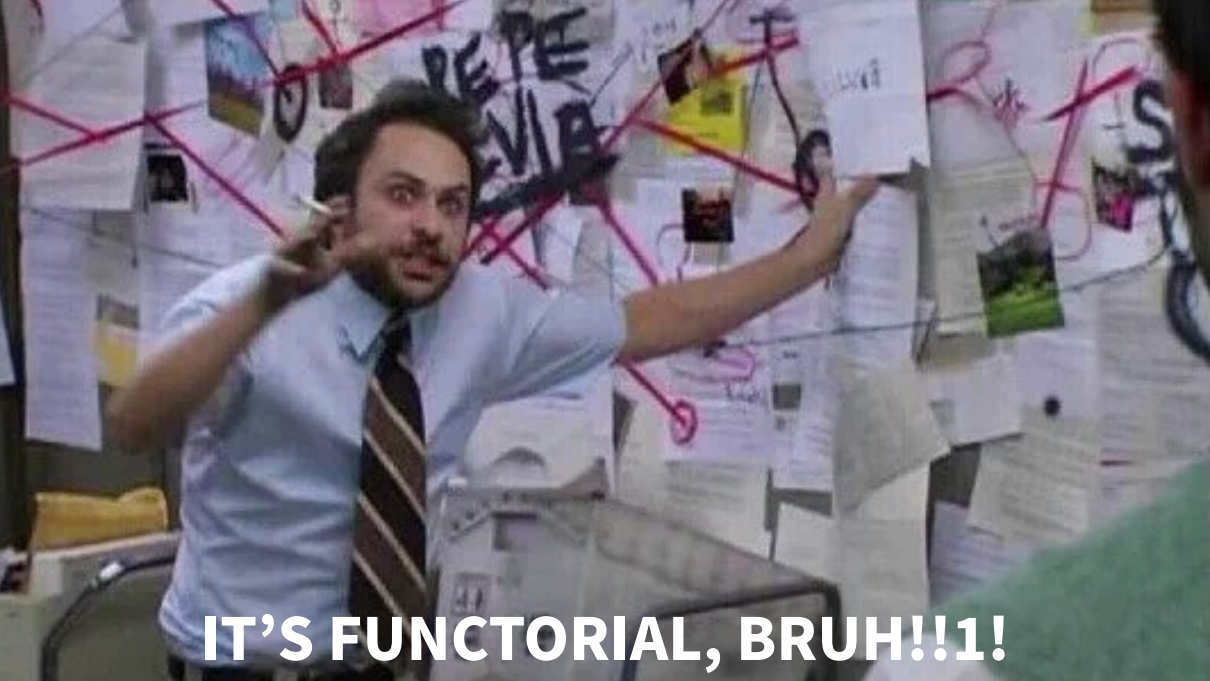

IT’S FUNCTORIAL, BRUH!!1!

Too small to fail

SQIsign v2 (June 2025)

	Sizes (bytes)		Timings (ms)		
Security	Public Key	Signature	Keygen	Sign	Verify
NIST-1	66	148	25	60	3
NIST-3	98	222	67	161	9
NIST-5	130	294	119	300	18

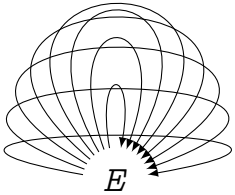
Technical slides ahead

Algebraic geometry: the classical PoV (…–1930's)

$$
\begin{array}{cccccc}
x^2 & +3y^2 & +xy & +10x & -7y & +4 = 0 \\
x^2 & -y^2 & +xy & +9x & -y & -3 = 0 \\
 & -2y^2 & & -3x & -8y & +9 = 0 \\
x^2 & +y^2 & +13xy & +10x & -7y & = 0 \\
x^2 & -5y^2 & +xy & +10x & +11y & +3 = 0 \\
x^2 & +y^2 & +11xy & -8x & +6y & = 0 \\
x^2 & & & & -9y & +4 = 0
\end{array}
$$

- Syzygies
  - Gröbner bases
  - Distinguishers for codes
- Invariants
  - Tensors
  - Exterior algebra
  - Moduli spaces
- …

Algebraic geometry: the modern PoV (1930's–…)



$$(\varphi + \psi)(P) := \varphi(P) + \psi(P)$$

$$(\varphi \circ \psi)(P) := \varphi(\psi(P))$$

$$\varphi \circ (\psi + \chi) = \varphi \circ \psi + \varphi \circ \chi$$

Endomorphism rings

$$\mathbb{Q}(\sqrt{-p})$$

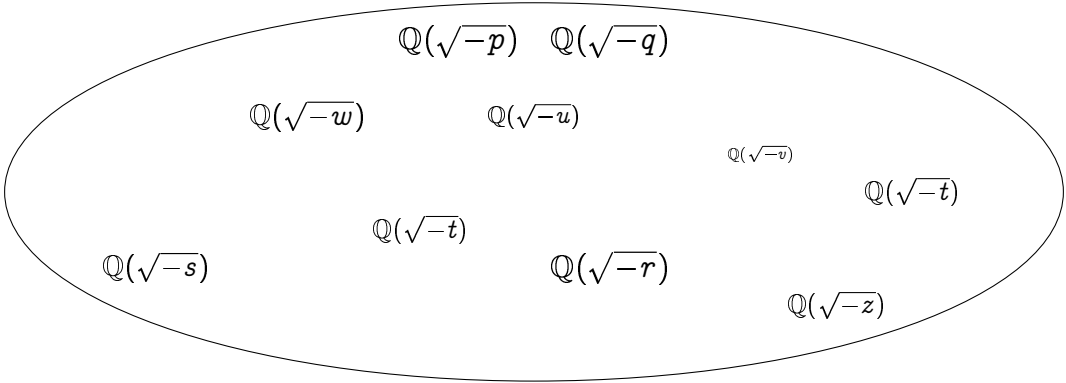
Quadratic imaginary field

Endomorphism rings

$$\mathbb{Q}(\sqrt{-p}) \quad \mathbb{Q}(\sqrt{-q})$$

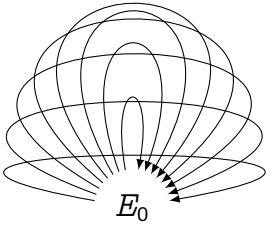
$$i^2 = -p \qquad j^2 = -q \qquad ij = -ji$$

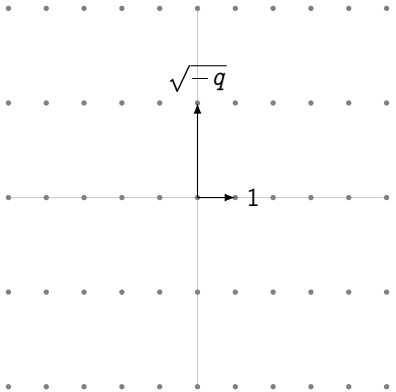
Endomorphism rings



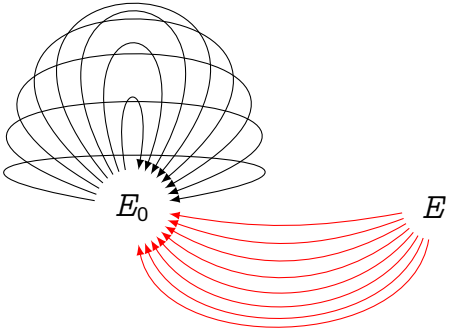
Quaternion algebra

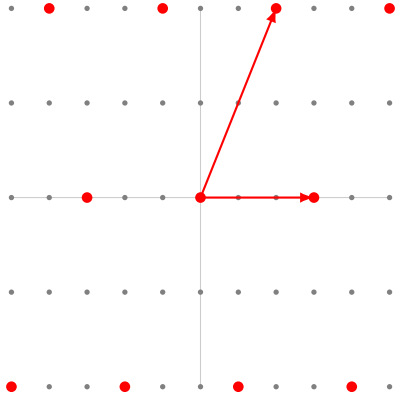
$$i^2 = -p \qquad j^2 = -q \qquad ij = -ji$$





$\mathbb{Z}[\sqrt{-q}]$





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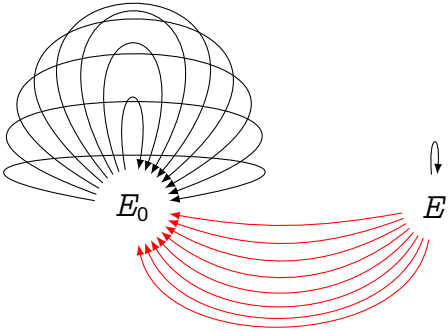
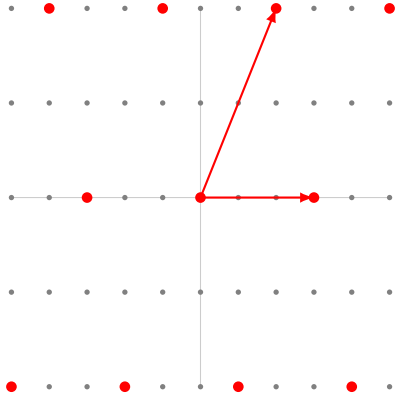
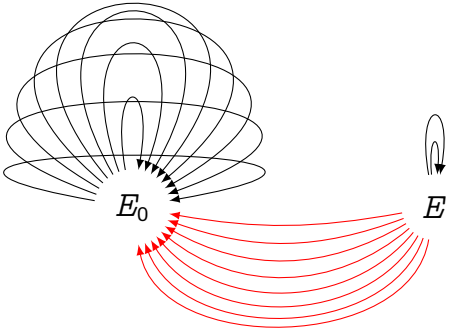
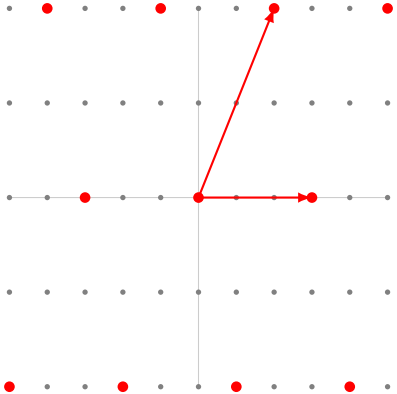




$\mathbb{Z}[\sqrt{-q}]$





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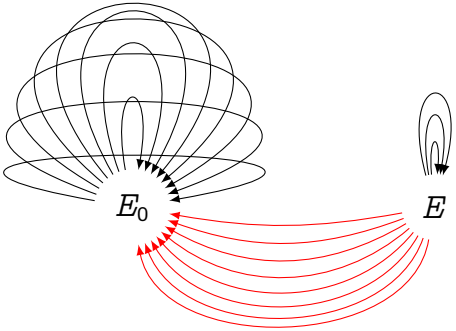

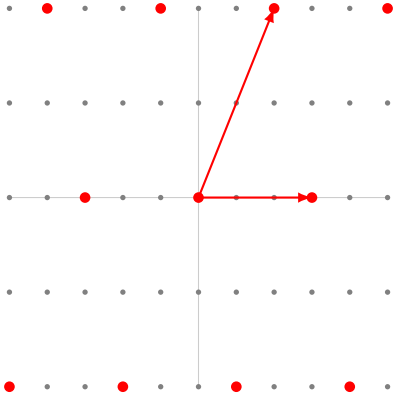

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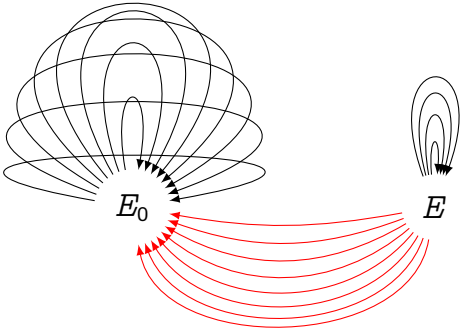

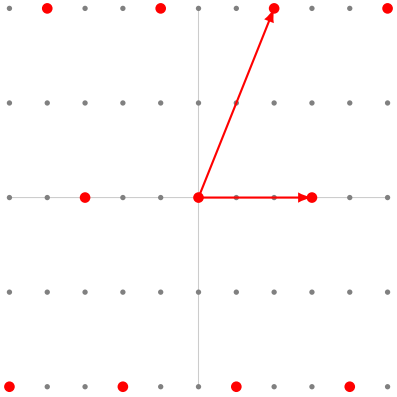

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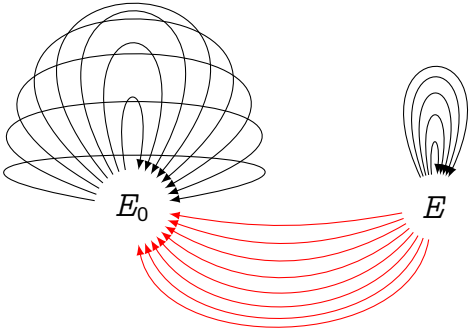

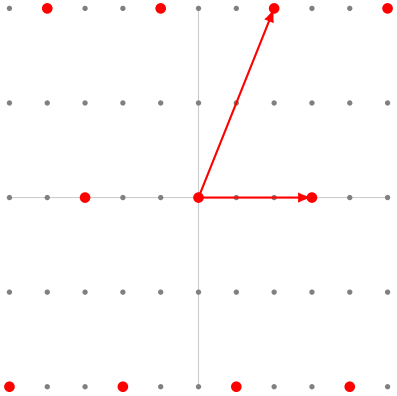

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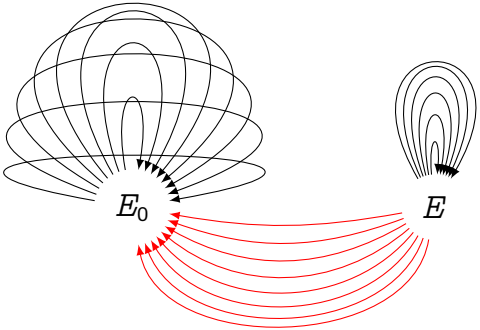

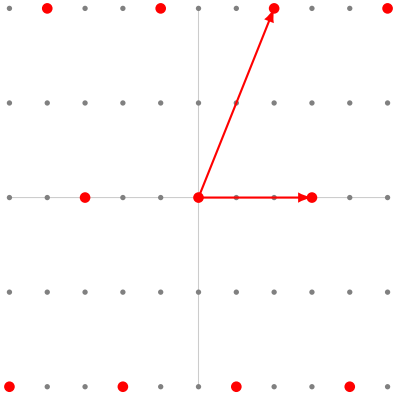

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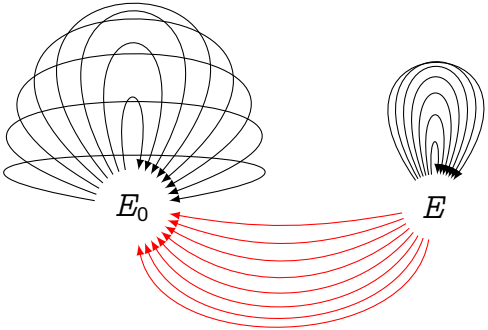

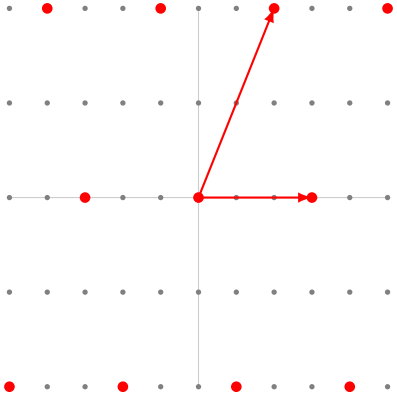

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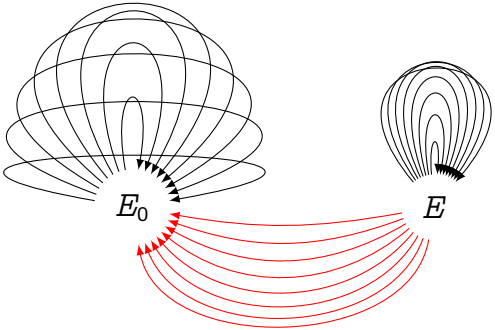

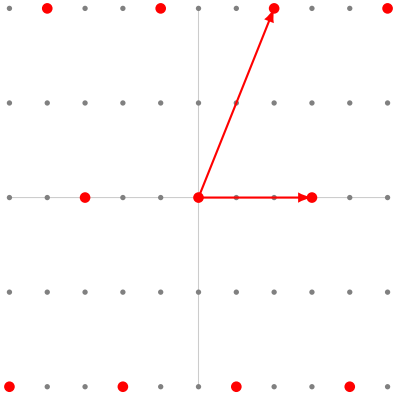

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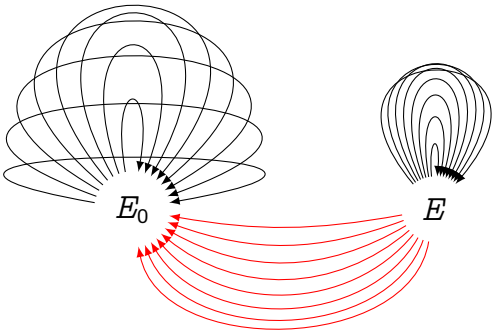

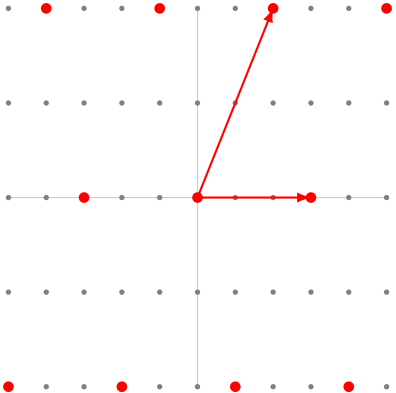

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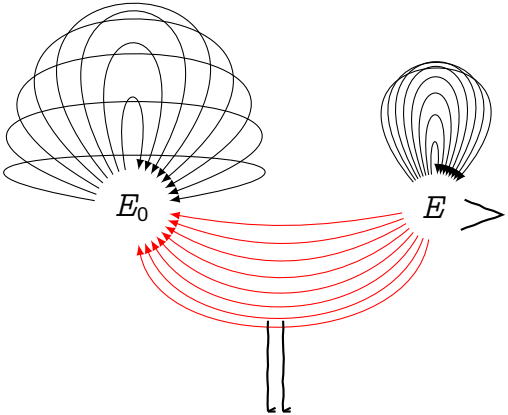
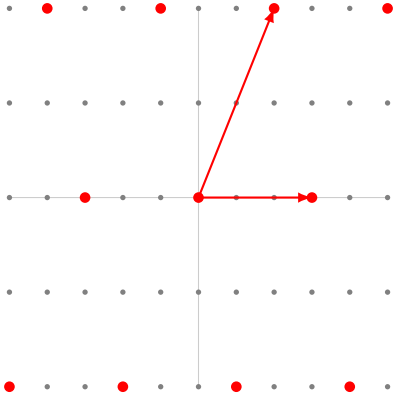
Deuring: $\mathrm{End}(E_0)$-lattices are in 1-to-1 correspondence with curves isogenous to E_0.





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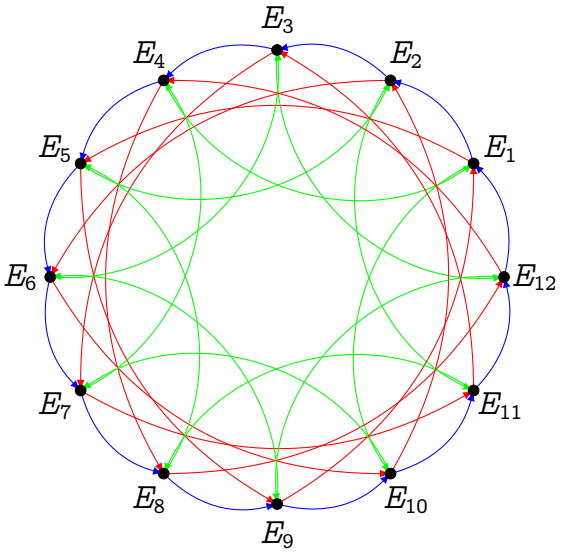
Class group action (Complex multiplication)



Lattices ↻ Isogenies

↓

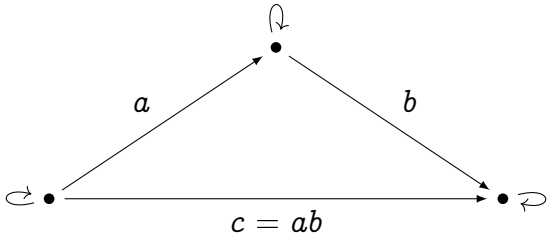
Ideals

↓

Ideal classes ↻ Curves

Grupoid action (Quaternionic multiplication)

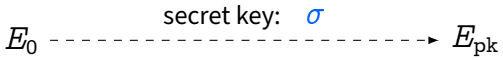
Grupoid = group with “partial” multiplication



(Ideal) Lattices  Curves

Algebra  Geometry

Identification Schemes from Group(oid) Actions



Graph isomorphism: Goldreich, Micali, Widgerson 1992;

Class group action: Couveignes 1997 / Stolbunov 2008;

Quaternionic multiplication: Galbraith, Petit, Silva 2016;

Code equivalence: Biasse, Micheli, Persichetti, Santini 2020;

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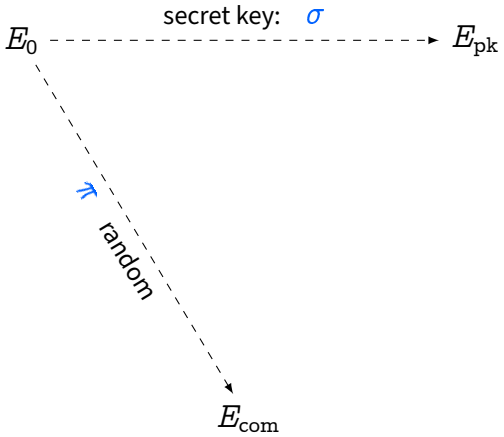

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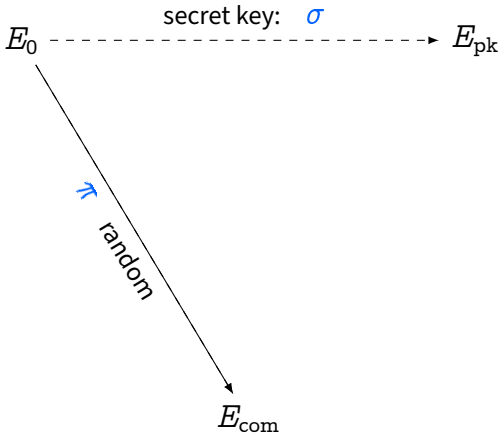

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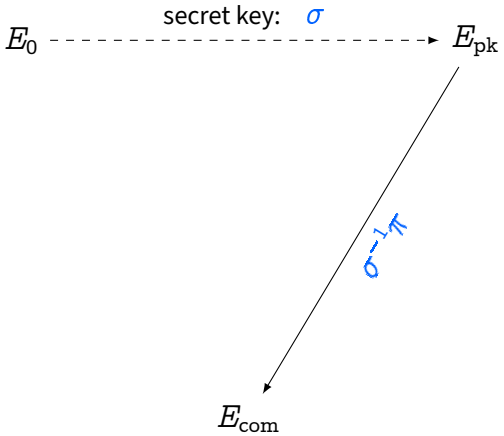

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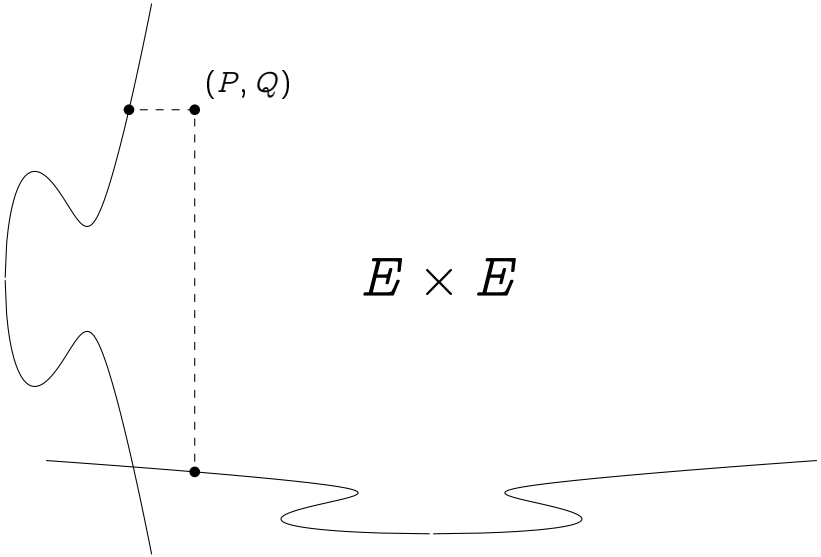
Coming soon

How general (or not) is SQIsign?

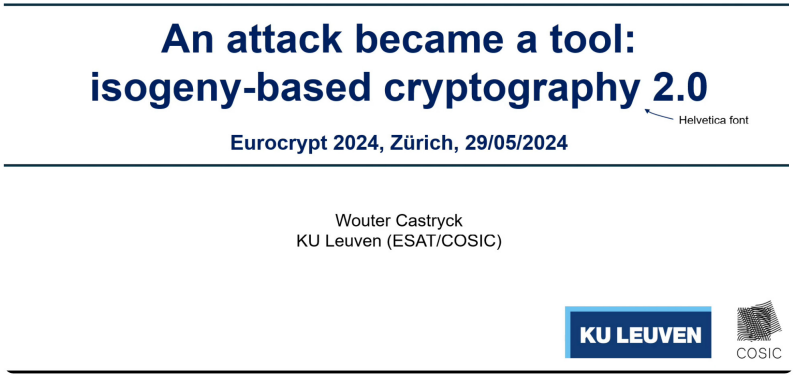
Beyond groupoids: axiomatizing SQIsign

joint work with Basso, Patranabis, Radulescu, Wesolowski

Higher dimensional abelian varieties



HD-mania



youtu.be/CpqjkfuXeoQ

Hermitian Lattices (rank g)

Abelian varieties (dimension g)

FAQ

Aren’t isogenies broken anyway?

As much as:

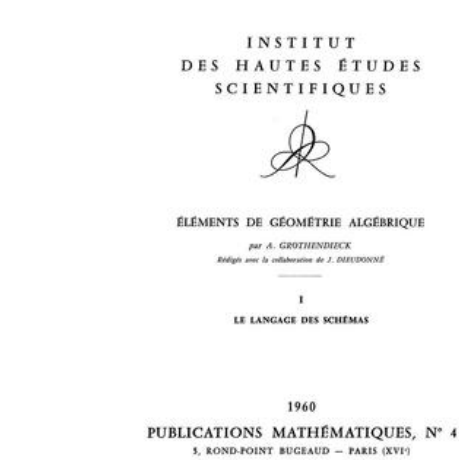
ECC: MOV pairing attack

Lattices: Basically any signature before Fiat–Shamir w aborts

Codes: Signatures, Syzygy distinguishers

Multivariate: HFE, Rainbow

Do I need to study the EGA to understand isogenies?



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1960
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No.

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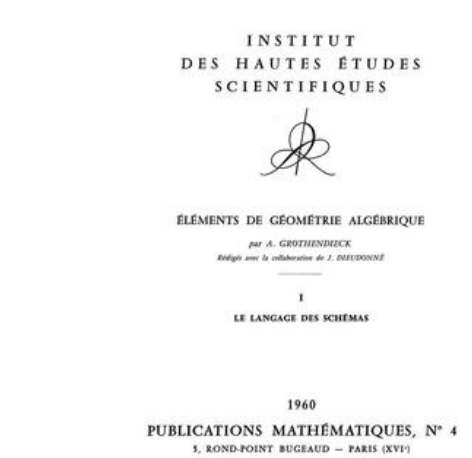

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No. But it won't hurt you.

When are you going to write a book?

When are you going to write a book?

Next question!

Community

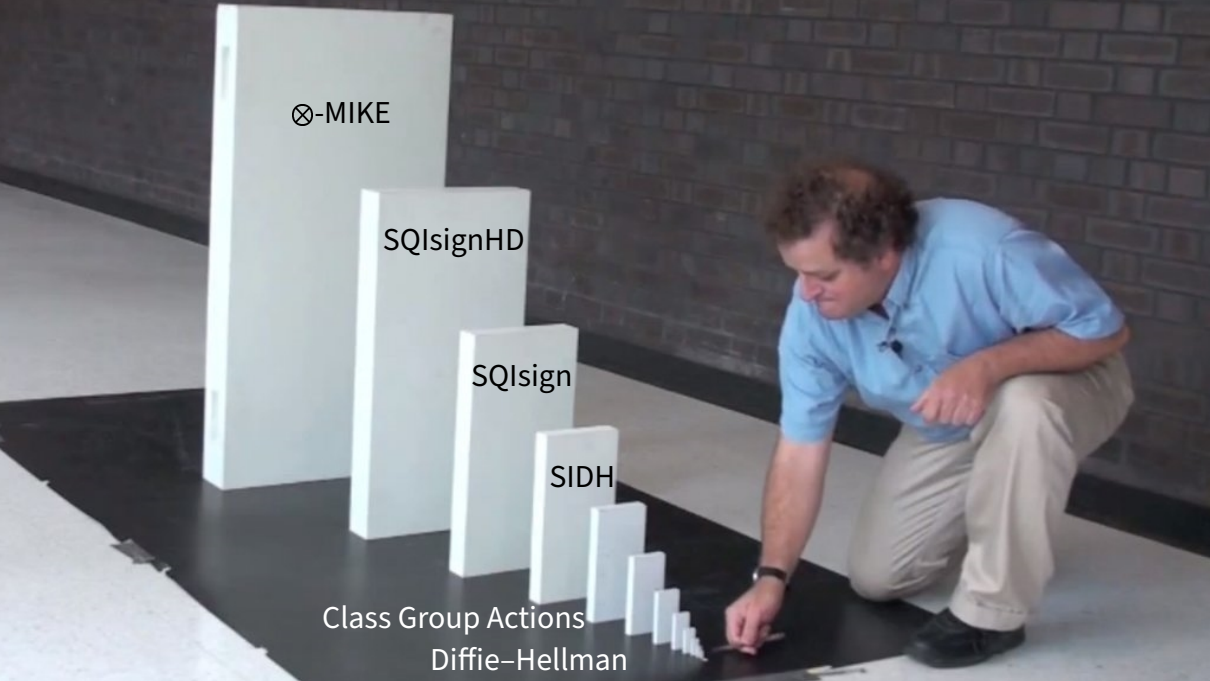
THE

isogeny club

https://isogeny.club

Famous last words

Algebra  Geometry


⊗-MIKE
SQIsignHD
SQIsign
SIDH
Class Group Actions
Diffie–Hellman

Thank you

https://defeo.lu/

@luca_defeo@ioc.exchange

@bsky.defeo.lu